**Nyenrode Business Universiteit**

**Answers of study exercises**
**Time Value of Money**

# Time Value of Money

## <u>Problem 1</u>

Happy Harry has just bought a scratch lottery ticket and won €10,000. He wants to finance the future study of his newly born daughter and invests this money in a fund with a maturity of 18 years offering a promising yearly return of 6%. What is the amount available on the 18th birthday of his daughter?

(1) Calculate <u>*future*</u> *value* or <u>present</u> value or <u>annuity</u> ?
(2) Future value = $PV * (1+ i)^n$

Items:

- PV = €10,000
- i = 6%
- n = 18 years

**Answer:**
**FV = €10,000 * $1.06^{18}$**
**= €10,000 * 2.854339 = €<u>28,543.39</u>**

## <u>Problem 2</u>

Rudy will retire in 20 years. This year he wants to fund an amount of €15,000 to become available in 20 years. How much does he have to deposit into a pension plan earning 7% annually?

(1) Calculate <u>future</u> value or <u>*present*</u> *value* or <u>annuity</u> ?
(2) Present value $= \frac{FV_n}{(1+i)^n}$

Items:

- FV = €15,000
- i = 7%
- n = 20 years

**Answer:**
**PV = €15,000 * ($1/1.07^{20}$)**
**= €15,000 * 0.258419 = €<u>3,876.29</u>**

## Problem 3

The National Savings Fund promises a monthly 0.75% return if you deposit €100 per month for 15 consecutive years. What amount will be accumulated after those 15 years?

(1) Calculate *future value* or present value or annuity?

(2) Future value = $A * \left[ \frac{(1+i)^n - 1}{i} \right]$

Items:

- Annuity = €100
- n = 15 years ???!!!
- i = 0.75% p/m

**Answer:**
**FVAN = €100 * (1.0075$^{180}$ – 1) / 0.0075**
**= €100 * 378.40577 = €37,840.58**

## Problem 4

Willy has just bought a house. She estimates that the roof will have to be renewed at a cost of €25,000 after 20 years. To cover these costs, she intends to save an equal amount of money at the end of each year, earning 6% annual interest rate. How much is such a yearly annuity?

(1) Calculate future value or present value or *annuity*?

(2) Future value = $A * \left[ \frac{(1+i)^n - 1}{i} \right]$

Items:

- FV = €25,000
- n = 20 years
- i = 6%

**Answer:**
**(FVAN)**
**€25,000 = A * ( 1.06$^{20}$ - 1 ) / 0.06 <=>**
**€25,000 = A * 36.7856 <=>**
**A = €25,000/ 36.7856 = €679.61**

## Problem 5

Pete considers buying a house. Currently, he rents a place for €1,000 a month. The current monthly interest rate on mortgages is 0.5%. His planning period is 20 years. If he doesn't want to increase his housing costs, what amount of mortgage is available for his purchase? *(Neglect any tax effects here).*

(1) Calculate <u>future</u> value or <u>*present*</u> value or <u>annuity</u>?

(2) Present value = $A * \left[ \dfrac{1 - \dfrac{1}{(1+i)^n}}{i} \right]$

Items:

- A = €1,000
- i = 0.5% per month
- n = 20 ??!

**Answer:**
**PVAN = €1,000 * [( 1 – (1/ $1.005^{240}$)) / 0.005]**
**= €1,000 * 139.5808 = <u>€139,580.80</u>**

## Problem 6

Liphips Ltd has just paid a dividend per share of €1.20. Shares are valued only on the basis of expected dividends. An annual sustainable growth of dividends of 4% is assumed. The appropriate discount rate **(i)** is 10% per year. The planning horizon is limited to 20 years. Compute the share value.

(1) Calculate <u>future</u> value or <u>*present*</u> value or <u>annuity</u>?

(2) Present value = $A * \left[ \dfrac{1 - \left( \dfrac{1+g}{1+i} \right)^n}{i - g} \right]$

Items:

- A = €1.20*1.04 = 1.248
- i = 10% annually
- g = 4% annually
- n = 20

**Answer:**
**(PVGRAN)**
**(1.20 * 1.04 ) * [1 – ($1.04^{20}$ / $1.1^{20}$) / (0.1 – 0.04)] => 1.248 * 11.238 = <u>14.03</u>**

## Problem 7

Compute the share value of a company paying a dividend of €3.60 per year over infinite maturity, with expected zero growth. The discount rate **(i)** is assumed to be 12% yearly.

(1) Calculate future value or *present value* or annuity?

(2) Present value = $\dfrac{A}{i}$

**Answer:**
**3.60 / 0.12 = 30**

## Problem 8

Centuries ago, rich families in the province of Friesland established a fund to further welfare and education. From this fund, only the interest revenue was allowed to be spent, in order to keep the principal unattached. Assume the fund has amounted to €12 million and market interest rate (i) is 6% annually. What would be the perpetuity (or present value of the fund) endowed to the society?

(1) Calculate future value or *present value* or annuity ?

(2) Present value $= \dfrac{A}{i}$

**Answer:**
**0.06 * €12 million / 0.06 = €12 million.**

## Problem 9

Calculate the value of a constant cash flow of €500 a year with a growth of 4%, measured over an infinite period at a discount rate (i) of 10%.

(1) Calculate future value or *present value* or annuity ?

(2) Present value = $A * \frac{1}{i - g}$

**Answer:**
**€500 * (1.04) / (0.10-0.04) = 8,666.67**

## Problem 10

Compute the effective yearly rate if the monthly rate is 1%.

$$i_e = \left(1 + \frac{i_n}{m}\right)^m - 1$$

**Answer:**
**- nominal (APR) => 1% * 12 = 12%.**
**- effective => ($1.01^{12}$)-1 (*100%) = 12.68%.**

## Problem 11

Compute the quarterly interest rate concerning an effective annual rate of 12% and a nominal annual rate of 12%.

**Answer:**
**Quarterly interest rate is: ($1.12^{(1/4)}$ -1) *100% = 2.87%**

*Additional:*
If the annual percentage rate is 12% concerning monthly compounding, what is then the effective annual rate?

$$i_e = \left(1 + \frac{APR}{m}\right)^m - 1$$

**Answer:**
**[1 + (12%/12)]$^{12}$ -1 =12.68%.**

What is then the effective rate for the first quarter?

**Answer:**
**$[1 + (12\%/12)]^3 - 1 = 3.03\%$.**

## Problem 12

A company buys a piece of equipment for €2 million on January 1. The expected useful life is 6 years and the salvage value is estimated zero. The company intends to replace the equipment identically. The average expected price increase is 8% yearly. For this purpose, the company creates a special fund with annual equal payments at the end of each year during the lifetime. Cost of capital and earnings of the fund (i) is 10% per year. Compute the annual payment into the fund.

(1) Calculate *future value* or present value or *annuity* ?

(2) Future value $= A * \left[ \frac{(1+i)^n - 1}{i} \right]$

**Answer:**
**Price at replacement time (or future value) => €2,000 x $1.08^6$ = €3,174**

**FVAN =>**
**€3,174 = A * $[(1.1)^6 - 1] / 0.1$ <=>**
**A = €3,174 / 7.71561 = €411.37**